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ABSTRACT: We examine five-dimensional $\mathcal{N} = 2$ gauged supergravity including terms up to four derivatives. These additional terms correspond to the supersymmetric completion of R^2, and were originally obtained in hep-th/0611329 using conformal supergravity techniques. Here we integrate out the auxiliary fields and obtain the on-shell action for minimal supergravity with such corrections. We then construct R-charged AdS black holes to linear order in the four derivative terms and investigate the effect of these corrections on their thermodynamical properties. Finally, we relate the geometrical coefficients governing the four-derivative corrections to gauge theory data using holographic anomaly matching. This enables us to obtain a microscopic expression for the entropy of the solutions.

Contents

1 Introduction

While one of the major achievements of modern physics has been the development of fundamental quantum field theories of matter, extending this to quantum gravity remains a challenge. In particular, conventional quantization of the Einstein-Hilbert action leads to a non-renormalizable theory. Nevertheless, both gravity and supergravity theories remain viable as effective field theories describing the low-energy limit of a UV complete theory such as string theory. Viewed in this light, it is then natural to explore higher derivative corrections to the two-derivative action.

Independent of supergravity, many people have considered higher derivative gravity theories such as $f(R)$ gravity, curvature-square theories, and so on. In terms of a derivative expansion, the first non-trivial terms enter at the R^2 level

$$e^{-1}\mathcal{L} = R + \alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \cdots . \tag{1.1}$$

In general, these additional terms modify the graviton propagator and give rise to ghosts (with the exception of the Gauss-Bonnet combination). While this was initially viewed as an argument against higher derivative gravity, these pathologies only show up at the

Planck scale, where traditional quantum gravity is already ill-defined due to its non-renormalizability. Furthermore, from the modern effective field theory point of view, such higher derivative terms are necessarily present, and carry information of the underlying UV complete theory.

A natural place to explore higher derivative supergravity theories is in the context of string theory, which gives rise to an effective low energy supergravity including higher derivative corrections. For example, it has been long known from string theory that the first curvature corrections to the Type II supergravity action appear at R^4 order [1–3], while corrections to heterotic supergravity first appear at R^2 order [4, 5]. Of course, even in the absence of stringy computations, supersymmetry itself puts strong constraints on the form of the higher derivative terms. Thus the absence of R^2 terms in Type II supergravity may also be viewed as a consequence of maximal supersymmetry. In general, the use of supersymmetry to constrain the form of the interactions is extremely powerful, and this is simply another example of this phenomenon.

In this paper, we investigate black holes in higher-derivative corrected five-dimensional $\mathcal{N} = 2$ gauged supergravity. Our motivation is two-fold. Firstly, we are interested in exploring the nature of stringy corrections to supergravity and in particular whether such higher-order corrections may smooth out singular horizons of small black holes. Secondly, five-dimensional gauged supergravity is a natural context in which to explore AdS/CFT, and black holes are important thermal backgrounds for this duality. By working out these gravity corrections, we may learn more about finite-coupling as well as $1/N$ effects in the dual $\mathcal{N} = 1$ super-Yang-Mills theory.

Because of the reduced supersymmetry, we expect the first corrections to $\mathcal{N} = 2$ gauged supergravity to occur at R^2 order. For this reason, we will limit our focus on four-derivative terms in the effective supergravity action. While in principle these terms may be derived directly from string theory, doing so would involve specific choices of string compactifications down to five dimensions as well as the potential need to work out contributions from the Ramond-Ramond sector. To avoid these issues, we instead make use of supersymmetry, and in particular the result of [6], which worked out the supersymmetric completion of the $A \wedge \mathrm{Tr}\, R \wedge R$ term in $\mathcal{N} = 2$ supergravity coupled to an arbitrary number of vector multiplets using the superconformal tensor calculus methods developed in [7–12].

Although we are not aware of an actual uniqueness proof, we expect the four-derivative terms constructed in [6] to be uniquely determined by supersymmetry (modulo field redefinitions). The ungauged story is rather elegant, and may be tied to M-theory compactified on a Calabi-Yau three-fold. In this case the higher derivative corrections are given by

$$e^{-1}\delta\mathcal{L} = \tfrac{1}{24}c_{2I}\left[\tfrac{1}{16}\epsilon_{\mu\nu\rho\lambda\sigma}A^{I\,\mu}R^{\nu\rho\alpha\beta}R^{\lambda\sigma}{}_{\alpha\beta} + \cdots\right], \tag{1.2}$$

where the ellipses denote the supersymmetric completion of the $A \wedge \mathrm{Tr}\, R \wedge R$ Chern-Simons term. Comparing this term with the Calabi-Yau reduction of the M5-brane anomaly term demonstrates that the coefficients c_{2I} are related to the second Chern class on the Calabi-Yau manifold. The higher-derivative corrected action has recently been applied to the study of five-dimensional black holes in string theory (see e.g. [13] and references therein).

While much has already been made of the higher-derivative corrections to ungauged supergravity, here we are mainly interested in the gauged supergravity case and resulting applications to AdS/CFT. In this case, the natural setup would be to take IIB string theory compactified on $\text{AdS}_5 \times Y^5$ where Y^5 is Sasaki-Einstein, which is dual to $\mathcal{N} = 1$ super-Yang-Mills theory in four dimensions. While the four-derivative terms worked out in [6] apply equally well to both gauged and ungauged supergravity, in this case their stringy origin is less clear. However, the c_{2I} coefficients governing the four-derivative terms may be related to gauge theory data using holographic anomaly matching [14–18], as we will see below.

Before constructing the R-charged black holes in the higher-derivative corrected theory, we first integrate out the auxiliary fields of the off-shell formulation (to linear order in c_{2I}), yielding an on-shell supergravity action. Throughout this paper, we furthermore work in the truncation to minimal supergravity involving only the graviton multiplet $(g_{\mu\nu}, A_\mu, \psi_\mu)$. While this on-shell action is implicit in the work of [6], we find it useful to have it written out explicitly, as it facilitates comparison with other recent results. This is especially of interest in providing a more rigorous supergravity understanding of the R^2 corrections to shear viscosity [19–21] and drag force [22, 23].

The outline of the paper is as follows. Section 2 is dedicated to obtaining the on-shell supergravity action. In section 3 we relate the gravitational parameters κ_5^2 and c_2 (the coefficients governing the four-derivative terms) to the central charges a, c of the dual CFT. In section 4 we construct static stationary R-charged AdS black holes with spherical, flat and hyperbolic ($k = 1, 0, -1$) horizons. These solutions, given to linear order in c_2, extend the well-known black hole solutions of the two-derivative theory [24, 25]. We also present a brief discussion on the effects of the higher derivative corrections on the structure of the horizon. Following this, in section 5 we study some basic thermodynamical properties of the black holes, including their temperature and entropy. We conclude in section 6 with a discussion.

2 Higher derivative gauged supergravity

In this section we investigate five-dimensional $\mathcal{N} = 2$ supergravity with the inclusion of (stringy) higher-derivative corrections. We are mainly interested in the case of gauged supergravity, which is the natural setting for the AdS/CFT setup. Because of the reduced amount of supersymmetry, we expect the first corrections to this theory to occur at R^2 order. For this reason, we will limit ourselves to four-derivative terms in the effective supergravity action.

The conventional on-shell formulation of minimal $\mathcal{N} = 2$ gauged supergravity [26, 27] is given in terms of the graviton multiplet $(g_{\mu\nu}, A_\mu, \psi_\mu^i)$ where ψ_μ^i is a symplectic-Majorana spinor with $i = 1, 2$ labeling the doublet of SU(2). The bosonic two-derivative Lagrangian takes the form

$$e^{-1}\mathcal{L}_0 = -R - \tfrac{1}{4}F_{\mu\nu}^2 + \tfrac{1}{12\sqrt{3}}\epsilon^{\mu\nu\rho\lambda\sigma}F_{\mu\nu}F_{\rho\lambda}A_\sigma + 12g^2, \tag{2.1}$$

where g is the coupling constant of the gauged R-symmetry, and where we have followed the sign conventions of [6].[1] We are, of course, interested in obtaining four-derivative correc-

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[1]We take $[\nabla_\mu, \nabla_\nu]v^\sigma = R_{\mu\nu\rho}{}^\sigma v^\rho$ and $R_{ab} = R_{ac}{}^c{}_b$.

tions to the above Lagrangian that are consistent with supersymmetry. Along with purely gravitational corrections of the form (1.1), other possible four-derivative terms include F^4, mixed RF^2 and parity violating ones. Given the large number of such terms, it would appear to be a daunting task to work out the appropriate supersymmetric combinations. Fortunately, however, it is possible to make use of manifest supersymmetry in the form of superconformal tensor calculus to construct supersymmetric R^2 terms. (See e.g. [28] for a nice review, albeit focusing on four-dimensional $\mathcal{N} = 2$ supergravity.)

The general idea of the superconformal approach[2] is to develop an off-shell formulation involving the Weyl multiplet that is locally gauge invariant under the superconformal group. The resulting conformal supergravity may then be broken down to Poincaré supergravity by introducing a conformal compensator in the hypermultiplet sector and introducing expectation values for some of its fields. One advantage of this method is that the off-shell formulation admits a superconformal tensor calculus which enables one to construct supersymmetric invariants of arbitrary order in curvature. This is in fact the approach taken in [6], which worked out the supersymmetric completion of the $A \wedge \operatorname{Tr} R \wedge R$ term in $\mathcal{N} = 2$ supergravity coupled to an arbitrary number of vector multiplets.

The basic construction of [6] involves conformal supergravity (i.e. the Weyl multiplet) coupled to a set of $n_V + 1$ conformal vector multiplets and a single compensator hypermultiplet. The resulting Lagrangian takes the form

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 = \mathcal{L}_0^{(V)} + \mathcal{L}_0^{(H)} + \mathcal{L}_1, \tag{2.2}$$

where $\mathcal{L}_0$ corresponds to the two-derivative terms and $\mathcal{L}_1$ the four-derivative terms. We have further broken up $\mathcal{L}_0$ into contributions $\mathcal{L}_0^{(V)}$ from the vector multiplets and $\mathcal{L}_0^{(H)}$ from the hypermultiplet.

As formulated in [6], the full Lagrangian $\mathcal{L}$ contains a set of auxiliary fields which we wish to eliminate in order to make direct comparison to the on-shell Lagrangian (2.1). To do so, we simply integrate out the auxiliary fields using their equations of motion, and the remainder of this section is devoted to this process. As an important shortcut, we note that when working to linear order in the correction terms in $\mathcal{L}_1$, we only need to substitute in the lowest order expressions for the auxiliary fields [33]. For this reason, we first examine the two-derivative Lagrangian before turning to the four-derivative terms contained in $\mathcal{L}_1$.

2.1 The leading two-derivative action

We begin with the vector multiplet contribution to the two-derivative Lagrangian [6]

$$e^{-1}\mathcal{L}_0^{(V)} = \mathcal{N}\left(\tfrac{1}{2}D - \tfrac{1}{4}R + 3v^2\right) + 2\mathcal{N}_I v^{\mu\nu}F^I_{\mu\nu} + \mathcal{N}_{IJ}\tfrac{1}{4}F^I_{\mu\nu}F^{J\,\mu\nu} + \tfrac{1}{24}c_{IJK}\epsilon^{\mu\nu\rho\lambda\sigma}A^I_\mu F^J_{\nu\rho}F^K_{\lambda\sigma} \\ -\mathcal{N}_{IJ}\left(\tfrac{1}{2}\mathcal{D}^\mu M^I \mathcal{D}_\mu M^J + Y^I_{ij}Y^{J\,ij}\right), \tag{2.3}$$

where M^I, A^I_μ and Y^I_{ij} ($I, J = 1, 2, \ldots, n_v + 1$) denote, respectively, the (real) scalar fields, the gauge fields and the $SU(2)$-triplet auxiliary fields in the $n_v + 1$ vector multiplets. In addition, the scalar D and the two-form $v_{\mu\nu}$ are auxiliary fields coming from the Weyl

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multiplet. The prepotential $\mathcal{N}$ and its functional derivatives are given by the standard expressions

$$\mathcal{N} = \tfrac{1}{6}c_{IJK}M^I M^J M^K, \qquad \mathcal{N}_I = \tfrac{1}{2}c_{IJK}M^J M^K, \qquad \mathcal{N}_{IJ} = c_{IJK}M^K. \qquad (2.4)$$

For future reference, we also note the useful relations

$$\mathcal{N}_I M^I = 3\mathcal{N}, \qquad \mathcal{N}_{IJ}M^J = 2\mathcal{N}_I. \qquad (2.5)$$

Turning next to the hypermultiplet Lagrangian, we have [6]

$$e^{-1}\mathcal{L}_0^{(H)} = 2\left[\mathcal{D}^\mu \mathcal{A}_i^{\bar\alpha}\mathcal{D}_\mu \mathcal{A}_\alpha^i + \mathcal{A}_i^{\bar\alpha}\,(g\,M)^2\,\mathcal{A}_\alpha^i + 2gY_{\alpha\beta}^{ij}\mathcal{A}_i^{\bar\alpha}\mathcal{A}_j^\beta\right] + \mathcal{A}^2\left(\tfrac{1}{4}D + \tfrac{3}{8}R - \tfrac{1}{2}v^2\right). \qquad (2.6)$$

In general, $\mathcal{A}_\alpha^i$ are a set of $4 \times n_H$ hypermatter scalars carrying both the SU(2) index i and the index $\alpha = 1, 2, \ldots, 2n_H$ of USp($2n_H$). (We use the SU(2) index raising convention $A^i = \epsilon^{ij}A_j$ and $A_i = A^j\epsilon_{ji}$ with $\epsilon_{12} = \epsilon^{12} = 1$). Note that we have gauged a subgroup G of USp($2n_H$), so that the covariant derivative appearing above is given by

$$\mathcal{D}_\mu \mathcal{A}_i^\alpha = \partial_\mu \mathcal{A}_i^\alpha - gA_\mu^I t_I \mathcal{A}_i^\alpha + \mathcal{A}_j^\alpha V_{\mu\,i}^j, \qquad (2.7)$$

where t_I are the generators of the gauge symmetry and where V_μ^{ij} is an additional auxiliary field belonging to the Weyl multiplet. Finally, we have defined $M \equiv M^I t_I$, where M^I are the vector multiplet scalars.

For simplicity, we focus on a single compensator and choose the conventional gauging of the diagonal U(1) in the SU(2) R-symmetry. In this case, the action of M on the hyperscalars is given by

$$M\mathcal{A}_i^\alpha = M^I t_I \mathcal{A}_i^\alpha = M^I P_I \left(i\sigma^3\right)_\beta^\alpha \mathcal{A}_i^\beta, \qquad (2.8)$$

while the covariant derivative becomes

$$\mathcal{D}_\mu \mathcal{A}_i^\alpha = \partial_\mu \mathcal{A}_i^\alpha - gA_\mu^I P_I \left(i\sigma^3\right)_\beta^\alpha \mathcal{A}_i^\beta + \mathcal{A}_j^\alpha V_{\mu\,i}^j. \qquad (2.9)$$

Here P_I denote the charges associated with the gauging. Furthermore, $\mathcal{A}^2 \equiv \mathcal{A}_i^{\bar\alpha}\mathcal{A}_\alpha^i = \mathcal{A}_i^\beta d_\beta{}^\alpha \mathcal{A}_\alpha^i$, where the metric $d_\beta{}^\alpha$ is arranged to be a Kronecker delta symbol as appropriate for a compensator [6].

Combining (2.3) with (2.6), the complete two-derivative action is given by

$$\begin{aligned}
e^{-1}\mathcal{L}_0 = {} & \tfrac{1}{4}D\left(2\mathcal{N} + \mathcal{A}^2\right) + R\left(\tfrac{3}{8}\mathcal{A}^2 - \tfrac{1}{4}\mathcal{N}\right) + v^2\left(3\mathcal{N} - \tfrac{1}{2}\mathcal{A}^2\right) \\
& + 2\mathcal{N}_I v^{\mu\nu}F_{\mu\nu}^I + \mathcal{N}_{IJ}\left(\tfrac{1}{4}F_{\mu\nu}^I F^{J\,\mu\nu} - \tfrac{1}{2}\mathcal{D}^\mu M^I \mathcal{D}_\mu M^J\right) + \tfrac{1}{24}c_{IJK}\epsilon^{\mu\nu\rho\lambda\sigma}A_\mu^I F_{\nu\rho}^J F_{\lambda\sigma}^K \\
& - \mathcal{N}_{IJ}Y_{ij}^I Y^{J\,ij} + 2\left[\mathcal{D}^\mu \mathcal{A}_i^{\bar\alpha}\mathcal{D}_\mu \mathcal{A}_\alpha^i + \mathcal{A}_i^{\bar\alpha}\,(g\,M)^2\,\mathcal{A}_\alpha^i + 2gY_{\alpha\beta}^{ij}\mathcal{A}_i^{\bar\alpha}\mathcal{A}_j^\beta\right]. \qquad (2.10)
\end{aligned}$$

At the two-derivative level, the auxiliary field D plays the role of a Lagrange multiplier, yielding the constraint

$$2\mathcal{N} + \mathcal{A}^2 = 0. \qquad (2.11)$$

Thus we can recover the standard very special geometry constraint $\mathcal{N} = 1$ by setting $\mathcal{A}^2 = -2$. (This fixing of the dilatational gauge transformation is in fact the purpose of the conformal compensator). This then brings the Lagrangian to the following form:

$$
\begin{aligned}
\mathcal{L}_0 = {}& \tfrac{1}{2}D(\mathcal{N} - 1) - \tfrac{1}{4}R(\mathcal{N} + 3) + v^2(3\mathcal{N} + 1) + 2\mathcal{N}_I v^{\mu\nu}F^I_{\mu\nu} \\
& + \mathcal{N}_{IJ}(\tfrac{1}{4}F^I_{\mu\nu}F^{J\,\mu\nu} - \tfrac{1}{2}\mathcal{D}^\mu M^I \mathcal{D}_\mu M^J) + \tfrac{1}{24}c_{IJK}\epsilon^{\mu\nu\rho\sigma}A^I_\mu F^J_{\nu\rho}F^K_{\lambda\sigma} \\
& - \mathcal{N}_{IJ}Y^I_{ij}Y^{J\,ij} + 2\left[\mathcal{D}^\mu \mathcal{A}^{\bar{\alpha}}_i \mathcal{D}_\mu \mathcal{A}^i_\alpha + \mathcal{A}^{\bar{\alpha}}_i (g\,M)^2 \mathcal{A}^i_\alpha + 2g Y^{ij}_{\alpha\beta}\mathcal{A}^{\bar{\alpha}}_i \mathcal{A}^\beta_j\right].
\end{aligned} \tag{2.12}
$$

2.1.1 Integrating out the auxiliary fields

The action (2.12) can be written in a more familiar on-shell form by integrating out the auxiliary fields. We will do this in two steps by first eliminating the fields $\mathcal{A}^\alpha_i$, V^{ij}_μ and Y^I_{ij} and then eliminating D and $v_{\mu\nu}$.

We start by fixing the SU(2) symmetry by taking $\mathcal{A}^\alpha_i = \delta^\alpha_i$, which identifies the indices in the hypermultiplet scalar. The equation of motion for V^{ij}_μ is then given by

$$
V^{ij}_\mu = g P_I \left(i\sigma^3\right)^{ij} A^I_\mu, \tag{2.13}
$$

which also results in $\mathcal{D}_\mu \mathcal{A}^\alpha_i = 0$. Turning next to Y^I_{ij}, we first note that

$$
Y^{ij}_{\alpha\beta}\mathcal{A}^{\bar{\alpha}}_i \mathcal{A}^\beta_j = g Y^{I\,ij}P_I \left(i\sigma^3\right)_{ij}. \tag{2.14}
$$

Varying (2.12) with respect to Y^I_{ij} then gives us the equation of motion

$$
Y^I_{ij} = 2\left(\mathcal{N}^{-1}\right)^{IJ}P_J \left(i\sigma^3\right)_{ij}. \tag{2.15}
$$

Using the above to eliminate $\mathcal{A}^\alpha_i$, V^{ij}_μ and Y^I_{ij} from the two-derivative action (2.12), we end up with

$$
\begin{aligned}
e^{-1}\mathcal{L}_0 = {}& \tfrac{1}{2}D(\mathcal{N} - 1) - \tfrac{1}{4}R(\mathcal{N} + 3) + v^2(3\mathcal{N} + 1) + 2\mathcal{N}_I v^{\mu\nu}F^I_{\mu\nu} \\
& + \mathcal{N}_{IJ}\left(\tfrac{1}{4}F^I_{\mu\nu}F^{J\,\mu\nu} - \tfrac{1}{2}\partial^\mu M^I \partial_\mu M^J\right) + \tfrac{1}{24}c_{IJK}\epsilon^{\mu\nu\rho\sigma}A^I_\mu F^J_{\nu\rho}F^K_{\lambda\sigma} \\
& + 8g^2\left(\mathcal{N}^{-1}\right)^{IJ}P_I P_J + 4g^2\left(P_I M^I\right)^2,
\end{aligned} \tag{2.16}
$$

where the last line corresponds to the gauged supergravity potential

$$
V = -4g^2\left[2\left(\mathcal{N}^{-1}\right)^{IJ}P_I P_J + \left(P_I M^I\right)^2\right]. \tag{2.17}
$$

Note that, with abelian gauging, the covariant derivative acts trivially on the vector multiplet scalars, $\mathcal{D}_\mu M^I = \partial_\mu M^I$.

To remove the remaining auxiliary fields D and $v_{\mu\nu}$ from (2.16) we must turn to the equations of motion for this system. Varying the action with respect to D, $v_{\mu\nu}$, M^I and A^I_μ yields, respectively,

$$
0 = \tfrac{1}{2}(\mathcal{N} - 1), \tag{2.18}
$$

$$
0 = 2(3\mathcal{N} + 1)v_{\mu\nu} + 2\mathcal{N}_I F^I_{\mu\nu}, \tag{2.19}
$$

$$
\begin{aligned}
0 = {}& \tfrac{1}{2}\mathcal{N}_I\left(D - \tfrac{1}{2}R + 6v_{\mu\nu}v^{\mu\nu}\right) + 2\mathcal{N}_{IJ}F^J_{\mu\nu}v^{\mu\nu} + \tfrac{1}{4}c_{IJK}F^J_{\mu\nu}F^{K\,\mu\nu} + \mathcal{N}_{IJ}\Box M^J \\
& + \tfrac{1}{2}c_{IJK}\partial_\mu M^J \partial^\mu M^K - \frac{\delta V}{\delta M^I},
\end{aligned} \tag{2.20}
$$

$$
0 = -\nabla^\nu\left[4\mathcal{N}_I v_{\nu\mu} + \mathcal{N}_{IJ}F^J_{\nu\mu}\right] + \tfrac{1}{8}C_{IJK}\epsilon_\mu{}^{\nu\rho\lambda\sigma}F^J_{\nu\rho}F^K_{\lambda\sigma}. \tag{2.21}
$$

In addition, the Einstein equation is given by:

$$0 = \tfrac{1}{4}(\mathcal{N}+3)\left(R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R\right)+\tfrac{1}{4}(\mathcal{N}-1)Dg_{\mu\nu}-\tfrac{1}{4}\left(\nabla_\mu\nabla_\nu\mathcal{N}-g_{\mu\nu}\Box\mathcal{N}\right)$$
$$+\tfrac{1}{2}\mathcal{N}_{IJ}\left(\partial_\mu M^I\partial_\nu M^J-\tfrac{1}{2}g_{\mu\nu}\partial_\lambda M^I\partial^\lambda M^J\right)-2(3\mathcal{N}+1)\left(v_{\mu\lambda}v_\nu{}^\lambda-\tfrac{1}{4}g_{\mu\nu}v_{\lambda\sigma}v^{\lambda\sigma}\right)$$
$$-4\mathcal{N}_I\left(F^I_{(\mu}{}^\lambda v_{\nu)\lambda}-\tfrac{1}{4}g_{\mu\nu}F^I_{\lambda\sigma}v^{\lambda\sigma}\right)-\tfrac{1}{2}\mathcal{N}_{IJ}\left(F^I_{\mu\lambda}F^{J\lambda}_\nu-\tfrac{1}{4}g_{\mu\nu}F^I_{\lambda\sigma}F^{J\lambda\sigma}\right)-\tfrac{1}{2}g_{\mu\nu}V\,. \quad (2.22)$$

We are now in a position to start solving for the auxiliary fields D and $v_{\mu\nu}$. Inserting the very special geometry constraint $\mathcal{N}=1$ (enforced by the equation of motion for D) into (2.19) yields

$$v_{\mu\nu}=-\tfrac{1}{4}\mathcal{N}_I F^I_{\mu\nu}\,. \quad (2.23)$$

We may now eliminate $\mathcal{N}$ and $v_{\mu\nu}$ from the lowest order Maxwell and Einstein equations to obtain

$$\nabla^\nu\left[(\mathcal{N}_I\mathcal{N}_J-\mathcal{N}_{IJ})\,F^J_{\nu\mu}\right]=-\tfrac{1}{8}C_{IJK}\epsilon_\mu{}^{\nu\rho\lambda\sigma}F^J_{\nu\rho}F^K_{\lambda\sigma},$$
$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R=-\tfrac{1}{2}\mathcal{N}_{IJ}\left(\partial_\mu M^I\partial_\nu M^J-\tfrac{1}{2}g_{\mu\nu}\partial_\lambda M^I\partial^\lambda M^J\right)$$
$$-\tfrac{1}{2}\left(\mathcal{N}_I\mathcal{N}_J-\mathcal{N}_{IJ}\right)\left(F^I_{\mu\lambda}F^{J\lambda}_\nu-\tfrac{1}{4}g_{\mu\nu}F^I_{\lambda\sigma}F^{J\lambda\sigma}\right)+\tfrac{1}{2}g_{\mu\nu}V\,. \quad (2.24)$$

Turning next to the scalar equations of motion, we note that the n_v+1 equations may be decomposed into n_v equations for the constrained scalars M^I, along with one equation for the Lagrange multiplier D. To solve for D, we multiply the scalar equation by M^I and obtain:

$$D-\tfrac{1}{2}R+6v_{\mu\nu}v^{\mu\nu}=-\tfrac{8}{3}\mathcal{N}_I F^I_{\mu\nu}v^{\mu\nu}-\tfrac{1}{6}\mathcal{N}_{IJ}F^I_{\mu\nu}F^{J\,\mu\nu}-\tfrac{1}{3}\mathcal{N}_{IJ}\partial_\mu M^I\partial^\mu M^J$$
$$-\tfrac{4}{3}\mathcal{N}_I\Box M^I+\tfrac{2}{3}M^I\frac{\delta V}{\delta M^I}\,. \quad (2.25)$$

Substituting in R and $v_{\mu\nu}$ then allows us to express the auxiliary field D entirely in terms of physical fields:

$$D=-\tfrac{7}{12}\mathcal{N}_{IJ}\partial_\mu M^I\partial^\mu M^J-\tfrac{4}{3}\mathcal{N}_I\Box M^I+\tfrac{1}{4}\left(\mathcal{N}_I\mathcal{N}_J-\tfrac{1}{2}\mathcal{N}_{IJ}\right)F^I_{\mu\nu}F^{J\,\mu\nu}-\tfrac{5}{6}V+\tfrac{2}{3}M^I\frac{\delta V}{\delta M^I}$$
$$=-\tfrac{7}{12}\mathcal{N}_{IJ}\partial_\mu M^I\partial^\mu M^J-\tfrac{4}{3}\mathcal{N}_I\Box M^I+\tfrac{1}{4}\left(\mathcal{N}_I\mathcal{N}_J-\tfrac{1}{2}\mathcal{N}_{IJ}\right)F^I_{\mu\nu}F^{J\,\mu\nu}$$
$$+2g^2\left[6P_IP_J\left(\mathcal{N}^{-1}\right)^{IJ}-P_IP_JM^IM^J\right]\,. \quad (2.26)$$

By using (2.25), the equation of motion for the constrained scalars (2.20) can be rewritten as

$$\left(\delta^J_I-\frac{\mathcal{N}_I M^J}{3}\right)\left[c_{JKL}\left(\partial_\mu M^K\partial^\mu M^L+2M^K\Box M^L\right)\right.$$
$$\left.-\left(\mathcal{N}_{JK}\mathcal{N}_L-\tfrac{1}{2}c_{JKL}\right)F^K_{\mu\nu}F^{L\,\mu\nu}-\frac{\delta V}{\delta M^J}\right]=0\,. \quad (2.27)$$

Note that the first term in parentheses acts as a projector, reenforcing the fact that the n_v+1 scalars M^I are constrained, so that there are only n_v independent scalar degrees of freedom.

We now have all the ingredients we need to write down the on-shell two-derivative Lagrangian:

$$e^{-1}\mathcal{L} = -R - \tfrac{1}{2}\mathcal{N}_{IJ}\partial_\mu M^I \partial^\mu M^J - \tfrac{1}{4}\left(\mathcal{N}_I\mathcal{N}_J - \mathcal{N}_{IJ}\right)F^I_{\mu\nu}F^{J\,\mu\nu}$$
$$+ \tfrac{1}{24}\,c_{IJK}\,\epsilon^{\mu\nu\rho\lambda\sigma}A^I_\mu F^J_{\nu\rho}F^K_{\lambda\sigma} + 4g^2\left[2\left(\mathcal{N}^{-1}\right)^{IJ}P_I P_J + \left(P_I M^I\right)^2\right], \quad (2.28)$$

where now the M^I are a set of constrained scalars satisfying the very special geometry condition $\mathcal{N} = 1$. Note that this Lagrangian may be brought into a more conventional form by defining the scalar metric

$$G_{IJ} = -\frac{1}{2}\frac{\partial}{\partial M^I}\frac{\partial}{\partial M^J}\mathcal{N}\bigg|_{\mathcal{N}=1} = -\frac{1}{2}\left(\mathcal{N}_{IJ} - \mathcal{N}_I\mathcal{N}_J\right), \quad (2.29)$$

with inverse

$$G^{IJ} = \left(G^{-1}\right)^{IJ} = M^I M^J - 2\left(\mathcal{N}^{-1}\right)^{IJ}. \quad (2.30)$$

In this case, (2.28) takes the form

$$e^{-1}\mathcal{L} = -R - \tfrac{1}{2}\mathcal{N}_{IJ}\partial_\mu M^I \partial^\mu M^J - \tfrac{1}{2}G_{IJ}F^I_{\mu\nu}F^{J\,\mu\nu} - V + \tfrac{1}{24}\,c_{IJK}\,\epsilon^{\mu\nu\rho\lambda\sigma}A^I_\mu F^J_{\nu\rho}F^K_{\lambda\sigma}, \quad (2.31)$$

where the potential is now given by

$$V = 4g^2\left(G^{IJ} - 2M^I M^J\right)P_I P_J. \quad (2.32)$$

This Lagrangian perfectly matches the bosonic sector of the standard two-derivative $\mathcal{N} = 2$ supergravity action coupled to n_v vector multiplets [26, 27, 34]. The resulting equations of motion are given by (2.24) and (2.27).

Here, we are mainly concerned with the truncation of (2.28) to the case of pure supergravity. This is accomplished by setting the scalars to constants[3] and by defining a single graviphoton A_μ according to[4]

$$M^I = \bar{M}^I, \qquad A^I_\mu = \bar{M}^I A_\mu. \quad (2.33)$$

While the constants $\bar{M}^I$ are arbitrary moduli in the ungauged case, in the gauged cause they must lie at a critical point of the potential (2.17) given by solving

$$\left(\delta^J_I - \frac{\mathcal{N}_I M^J}{3}\right)\frac{\delta V}{\delta M^J} = 0. \quad (2.34)$$

By demanding that the critical point is supersymmetric, we find that the constant scalars satisfy:[5]

$$P_I \bar{M}^I = \frac{3}{2}, \qquad \left(\bar{\mathcal{N}}^{-1}\right)^{IJ}P_I P_J = \frac{3}{8}. \quad (2.35)$$

---

[3]Recall that the $D = 5$, $\mathcal{N} = 2$ scalars M^I are real. The barred notation $\bar{M}^I$ is simply used to denote the constant values of the scalars and related quantities at this order.

[4]Note that our definition differs by a factor of $1/3$ from the conventional one where $A_\mu = A^I_\mu \mathcal{N}_I$.

[5]These expressions can be obtained by making use of the hyperino and gauging SUSY variations, as well as the equation of motion for the auxiliary field Y^I_{ij}. We refer the reader to [6] for more details.

in which case the potential becomes $\bar{V} = -12g^2$. The resulting Lagrangian for the bosonic fields of the supergravity multiplet $(g_{\mu\nu}, A_\mu)$ then reads

$$e^{-1}\mathcal{L} = -R - \tfrac{3}{4}F_{\mu\nu}^2 + \tfrac{1}{4}\epsilon^{\mu\nu\rho\lambda\sigma}A_\mu F_{\nu\rho}F_{\lambda\sigma} + 12g^2 \,, \tag{2.36}$$

which reproduces the conventional on-shell supergravity Lagrangian (2.1) once the graviphoton is rescaled according to $A_\mu \to A_\mu/\sqrt{3}$.

While this completes the analysis relevant to the leading, two-derivative action, we note that the expression for D simplifies further in the case of constant scalars. Substituting (2.33) and (2.35) into the expression (2.26) for D yields the simple result

$$D = \tfrac{1}{4}\left(\bar{\mathcal{N}}_I\bar{\mathcal{N}}_J - \tfrac{1}{2}\bar{\mathcal{N}}_{IJ}\right)F_{\mu\nu}^I F^{J\,\mu\nu} = \tfrac{3}{2}F_{\mu\nu}^2 \,. \tag{2.37}$$

By taking $\mathcal{N} = 1$, we see that this explicit form of D does not play a role in the leading expression for the two-derivative Lagrangian. However, it will become relevant in the discussion of higher derivative corrections, which we turn to next.

2.2 Higher-derivative corrections in gauged SUGRA

We now turn to the four-derivative corrections to the action (2.2), which we parameterize by $\mathcal{L}_1$. For convenience, we separate the contributions to $\mathcal{L}_1$ present in the ungauged theory from those coming strictly from the gauging, $\mathcal{L}_1 = \mathcal{L}_1^{\text{ungauged}} + \mathcal{L}_1^{\text{gauged}}$. The two are given by:

$$
\begin{aligned}
e^{-1}\mathcal{L}_1^{\text{ungauged}} = {}&\tfrac{1}{24}c_{2I}\Big[\tfrac{1}{16}\epsilon_{\mu\nu\rho\lambda\sigma}A^{I\,\mu}R^{\nu\rho\alpha\beta}R^{\lambda\sigma}{}_{\alpha\beta} + \tfrac{1}{8}M^I C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \tfrac{1}{12}M^I D^2 + \tfrac{1}{6}F_{\mu\nu}^I v^{\mu\nu}D \\
&-\tfrac{1}{3}M^I C_{\mu\nu\rho\sigma}v^{\mu\nu}v^{\rho\sigma} - \tfrac{1}{2}F^{I\,\mu\nu}C_{\mu\nu\rho\sigma}v^{\rho\sigma} + \tfrac{8}{3}M^I v_{\mu\nu}\nabla^\nu\nabla_\rho v^{\mu\rho} \\
&-\tfrac{16}{9}M^I v^{\mu\rho}v_{\rho\nu}R^\nu_\mu - \tfrac{2}{9}M^I v^2 R + \tfrac{4}{3}M^I \nabla^\mu v^{\nu\rho}\nabla_\mu v_{\nu\rho} + \tfrac{4}{3}M^I \nabla^\mu v^{\nu\rho}\nabla_\nu v_{\rho\mu} \\
&-\tfrac{2}{3}M^I \epsilon_{\mu\nu\rho\lambda\sigma}v^{\mu\nu}v^{\rho\lambda}\nabla_\delta v^{\sigma\delta} + \tfrac{2}{3}F^{I\,\mu\nu}\epsilon_{\mu\nu\rho\lambda\sigma}v^{\rho\delta}\nabla_\delta v^{\lambda\sigma} + F^{I\,\mu\nu}\epsilon_{\mu\nu\rho\lambda\sigma}v^\rho{}_\delta\nabla^\lambda v^{\sigma\delta} \\
&-\tfrac{4}{3}F^{I\,\mu\nu}v_{\mu\rho}v^{\rho\lambda}v_{\lambda\nu} - \tfrac{1}{3}F^{I\,\mu\nu}v_{\mu\nu}v^2 + 4M^I v_{\mu\nu}v^{\nu\rho}v_{\rho\lambda}v^{\lambda\mu} - M^I\left(v^2\right)^2\Big], \quad (2.38)
\end{aligned}
$$

$$
\begin{aligned}
e^{-1}\mathcal{L}_1^{\text{gauged}} = {}&\tfrac{1}{24}c_{2I}\Big[-\tfrac{1}{12}\epsilon_{\mu\nu\rho\lambda\sigma}\,A^{I\,\mu}R^{\nu\rho\,ij}(U)R^{\lambda\sigma}_{ij}(U) \\
&-\tfrac{1}{3}M^I R^{\mu\nu\,ij}(U)R_{\mu\nu\,ij}(U) - \tfrac{4}{3}Y^I_{ij}v_{\mu\nu}R^{\mu\nu\,ij}(U)\Big], \quad (2.39)
\end{aligned}
$$

where

$$R_{\mu\nu}{}^{ij}(U) = \partial_\mu V_\nu^{ij} - V_{\mu k}^i V_\nu^{kj} - (\mu \leftrightarrow \nu)\,. \tag{2.40}$$

As we can see, the constants c_{2I} parameterize the magnitude of these contributions. Notice that the scalar D no longer acts as a Lagrange multiplier, since it now appears quadratically in $\mathcal{L}_1$. In fact, by varying the full action $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$ with respect to D, with $\mathcal{L}_0$ as in (2.16), we obtain the modified very special geometry constraint

$$\mathcal{N} = 1 - \frac{c_{2I}}{72}\left(DM^I + F^{I\,\mu\nu}v_{\mu\nu}\right)\,, \tag{2.41}$$

which encodes information about how the scalars M^I are affected by higher-derivative corrections.

2.2.1 Integrating out the auxiliary fields

As in the two-derivative case, in order to obtain a Lagrangian written solely in terms of the physical fields of the theory we need to eliminate the auxiliary fields D, $v_{\mu\nu}$, $V_{\mu\nu}^i$ and Y_{ij}^I from $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$. In section 2.1 we solved for the auxiliary fields by neglecting higher order corrections, and then integrated them out of the two-derivative action. It turns out that the lowest order expressions for the auxiliary fields are sufficient when working to linear order in the c_{2I} [33]. This allows us to reuse the results of the previous section for the auxiliary fields, which we summarize here:

$$V_{\mu}^{ij} = gP_I \left(i\sigma^3\right)^{ij} A_{\mu}^I \,, \tag{2.42}$$

$$Y_{ij}^I = 2\left(\mathcal{N}^{-1}\right)^{IJ} P_J \left(i\sigma^3\right)_{ij} \,, \tag{2.43}$$

$$v_{\mu\nu} = -\tfrac{1}{4}\mathcal{N}_I F_{\mu\nu}^I \,, \tag{2.44}$$

$$D = \tfrac{1}{4}\left(\mathcal{N}_I\mathcal{N}_J - \tfrac{1}{2}\mathcal{N}_{IJ}\right) F_{\mu\nu}^I F^{J\,\mu\nu} \,. \tag{2.45}$$

While it is valid to use these lowest order expressions, it is important to realize that the scalar fields are modified because of (2.41). This modification leads to additional contributions to the two-derivative on-shell action (2.28), which combines with $\mathcal{L}_1$ to yield the complete action at linear order in c_{2I}.

In principle, we may work with the full system of supergravity coupled to n_V vector multiplets. However, here we focus on the truncation to pure supergravity, where the scalars M^I are taken to be non-dynamical. Even so, they are not entirely trivial. While at the two-derivative level, we may simply set them to constants according to (2.33), here we must allow for the modification (2.41) by defining

$$M^I = \bar{M}^I + c_2\hat{M}^I, \qquad A_{\mu}^I = \bar{M}^I A_{\mu}, \qquad c_2 \equiv c_{2I}\bar{M}^I \,, \tag{2.46}$$

where $\hat{M}^I$ are possible scalar fluctuations that enter at $\mathcal{O}(c_2)$. Substituting this into the expressions (2.44) and (2.45) for the auxiliary fields then yields

$$v_{\mu\nu} = -\tfrac{3}{4}F_{\mu\nu} + \mathcal{O}\left(c_2\right), \qquad D = \tfrac{3}{2}F^2 + \mathcal{O}\left(c_2\right) \,, \tag{2.47}$$

which match the lowest order expressions for constant scalars. The modified very special geometry constraint (2.41) can now be simplified further, and becomes

$$\mathcal{N} = 1 - \frac{c_2}{96}F^2 + \mathcal{O}\left(c_2^2\right) \,. \tag{2.48}$$

The most general solution for the fluctuating scalars $\hat{M}^I$ ought to come from solving the equations of motion. However, as a shortcut, we make the *ansatz* that $\hat{M}^I$ is proportional to $\bar{M}^I$. The modified constraint (2.48) is then enough to fix the correction to the scalars to be

$$M^I = \bar{M}^I \left[1 - \frac{c_2}{288}F^2 + \mathcal{O}\left(c_2^2\right)\right] \,. \tag{2.49}$$

Consistency of this ansatz with the equations of motion also demands an appropriate relation between the various c_{2I} coefficients. Finding this relation involves solving for the

auxiliary field D to linear order in c_{2I}. When applied to the M^I equation of motion this imposes the condition

$$c_{2I} = \frac{1}{3} c_{2J} \bar{M}^J \bar{\mathcal{N}}_I = \frac{1}{3} c_2 \bar{\mathcal{N}}_I. \tag{2.50}$$

Although this appears to place a strong restriction on the form of the higher derivative corrections, since we are only interested in the system where the vector multiplets are truncated out, it makes no difference what the conditions were prior to truncation, so long as the combination of parameters c_2 given in (2.46) survives.

We are now ready to integrate out both the scalars M^I and the auxiliary fields from the two-derivative action $\mathcal{L}_0$ given in (2.12). By making use of the corrections[6] to the leading order scalar expressions (2.35)

$$P_I M^I = \frac{3}{2} \left[1 - \frac{c_2}{288} F^2 \right], \qquad \left(\mathcal{N}^{-1} \right)^{IJ} P_I P_J = \frac{3}{8} \left[1 + \frac{c_2}{288} F^2 \right], \tag{2.51}$$

we find that the contribution coming from $\mathcal{L}_0$ yields the following terms:

$$e^{-1} \mathcal{L}_0 = -R - \frac{3}{4} F^2 + \frac{1}{4} \epsilon^{\mu\nu\rho\lambda\sigma} A_\mu F_{\nu\rho} F_{\lambda\sigma} + 12 g^2 + \frac{c_2}{24} \left[\frac{1}{16} R F^2 + \frac{1}{64} \left(F^2 \right)^2 - \frac{5}{4} g^2 F^2 \right]. \tag{2.52}$$

Turning next to the four-derivative contributions, we note that, since such terms are already linear in c_2, we may simply use the leading order solution for the scalars. The gauging contribution (2.39) is then particularly simple

$$e^{-1} \mathcal{L}_1^{\text{gauged}} = -\frac{c_2}{64} g^2 \, \epsilon_{\mu\nu\rho\lambda\sigma} A^\mu F^{\nu\rho} F^{\lambda\sigma} . \tag{2.53}$$

On the other hand, the contribution to $\mathcal{L}_1^{\text{ungauged}}$ is given by:

$$\begin{aligned}
e^{-1} \mathcal{L}_1^{\text{ungauged}} = \frac{c_2}{24} \Big[& \frac{1}{16} \, \epsilon_{\mu\nu\rho\lambda\sigma} A^\mu R^{\nu\rho\delta\gamma} R^{\lambda\sigma}{}_{\delta\gamma} + \frac{1}{8} C^2_{\mu\nu\rho\sigma} + \frac{3}{16} C_{\mu\nu\rho\lambda} F^{\mu\nu} F^{\rho\lambda} - F^{\mu\rho} F_{\rho\nu} R^\nu_\mu \\
& - \frac{1}{8} R F^2 + \frac{3}{2} F_{\mu\nu} \nabla^\nu \nabla_\rho F^{\mu\rho} + \frac{3}{4} \nabla^\mu F^{\nu\rho} \nabla_\mu F_{\nu\rho} + \frac{3}{4} \nabla^\mu F^{\nu\rho} \nabla_\nu F_{\rho\mu} \\
& + \frac{1}{8} \, \epsilon_{\mu\nu\rho\lambda\sigma} F^{\mu\nu} (3 F^{\rho\lambda} \nabla_\delta F^{\sigma\delta} + 4 F^{\rho\delta} \nabla_\delta F^{\lambda\sigma} + 6 F^\rho{}_\delta \nabla^\lambda F^{\sigma\delta}) \\
& + \frac{45}{64} F_{\mu\nu} F^{\nu\rho} F_{\rho\lambda} F^{\lambda\mu} - \frac{45}{256} \left(F^2 \right)^2 \Big] . \end{aligned} \tag{2.54}$$

The full on-shell Lagrangian is thus given by

$$\begin{aligned}
e^{-1} \mathcal{L} = -R - \frac{3}{4} F^2 \left(1 + \frac{5}{72} c_2 g^2 \right) + \frac{1}{4} \left(1 - \frac{1}{16} c_2 g^2 \right) \epsilon^{\mu\nu\rho\lambda\sigma} A_\mu F_{\nu\rho} F_{\lambda\sigma} + 12 g^2 \\
+ \frac{c_2}{24} \left[\frac{1}{16} R F^2 + \frac{1}{64} \left(F^2 \right)^2 \right] + \mathcal{L}_1^{\text{ungauged}} . \end{aligned} \tag{2.55}$$

Finally, we may redefine A_μ to write the kinetic term in canonical form:

$$A_\mu^{\text{final}} = \sqrt{3} \left(1 + \frac{5}{144} c_2 g^2 \right) A_\mu^{\text{old}} . \tag{2.56}$$

---

[6]These can be easily verified using $P_I = \frac{1}{4} \bar{\mathcal{N}}_{IJ} \bar{M}^J$.

The Lagrangian then becomes:

$$\mathcal{L} = -R - \frac{1}{4}F^2 + \frac{1}{12\sqrt{3}}\left(1 - \frac{1}{6}c_2 g^2\right)\epsilon^{\mu\nu\rho\lambda\sigma}A_\mu F_{\nu\rho}F_{\lambda\sigma} + 12g^2$$
$$+ \frac{c_2}{24}\left[\frac{1}{48}RF^2 + \frac{1}{576}\left(F^2\right)^2\right] + \mathcal{L}_1^{\text{ungauged}}, \tag{2.57}$$

with

$$e^{-1}\mathcal{L}_1^{\text{ungauged}} = \frac{c_2}{24}\Big[\frac{1}{16\sqrt{3}}\epsilon_{\mu\nu\rho\lambda\sigma}A^\mu R^{\nu\rho\delta\gamma}R^{\lambda\sigma}{}_{\delta\gamma} + \frac{1}{8}C^2_{\mu\nu\rho\sigma} + \frac{1}{16}C_{\mu\nu\rho\lambda}F^{\mu\nu}F^{\rho\lambda} - \frac{1}{3}F^{\mu\rho}F_{\rho\nu}R^\nu_\mu$$
$$-\frac{1}{24}RF^2 + \frac{1}{2}F_{\mu\nu}\nabla^\nu\nabla_\rho F^{\mu\rho} + \frac{1}{4}\nabla^\mu F^{\nu\rho}\nabla_\mu F_{\nu\rho} + \frac{1}{4}\nabla^\mu F^{\nu\rho}\nabla_\nu F_{\rho\mu}$$
$$+\frac{1}{32\sqrt{3}}\epsilon_{\mu\nu\rho\lambda\sigma}F^{\mu\nu}\left(3F^{\rho\lambda}\nabla_\delta F^{\sigma\delta} + 4F^{\rho\delta}\nabla_\delta F^{\lambda\sigma} + 6F^\rho{}_\delta\nabla^\lambda F^{\sigma\delta}\right)$$
$$+\frac{5}{64}F_{\mu\nu}F^{\nu\rho}F_{\rho\lambda}F^{\lambda\mu} - \frac{5}{256}\left(F^2\right)^2\Big]. \tag{2.58}$$

3 Anomaly matching and AdS/CFT

In the above section, we have written out the on-shell five-dimensional $\mathcal{N} = 2$ gauged supergravity Lagrangian up to four-derivative order. Restoring Newton's constant, this takes the form

$$e^{-1}\mathcal{L} = \frac{1}{16\pi G_5}\left[-R - \frac{1}{4}F^2 + \frac{1}{12\sqrt{3}}\epsilon^{\mu\nu\rho\lambda\sigma}A_\mu F_{\nu\rho}F_{\lambda\sigma} + 12g^2 + \frac{c_2}{192}C^2_{\mu\nu\rho\sigma} + \cdots\right], \tag{3.1}$$

where we have only written out a few noteworthy terms. Given this Lagrangian, it is natural to make the appropriate AdS/CFT connection to $\mathcal{N} = 1$ super-Yang Mills theory. Before we do so, however, we present a brief review of the AdS/CFT dictionary in the case of $\mathcal{N} = 4$ super-Yang Mills.

The standard AdS/CFT setup relates IIB string theory on $\text{AdS}_5 \times S^5$ to $\mathcal{N} = 4$ super-Yang Mills with gauge group $\text{SU}(N)$ and 't Hooft coupling $\lambda = g_{\text{YM}}^2 N$. The standard AdS/CFT dictionary then reads

$$\frac{L^4}{\alpha'^2} = 4\pi g_s N = g_{\text{YM}}^2 N, \tag{3.2}$$

where L is the 'radius' of AdS_5. This duality may be approached more directly by reducing IIB supergravity on S^5, yielding $\mathcal{N} = 8$ gauged supergravity in five dimensions. Just as in the $\mathcal{N} = 2$ case of (3.1), this theory is determined in terms of two gravity-side parameters, G_5 (Newton's constant) and g (the gauged supergravity coupling constant). These are related to the parameters of the AdS/CFT dictionary (3.2) according to

$$g = \frac{1}{L}, \qquad N^2 = \frac{\pi L^3}{2G_5}. \tag{3.3}$$

Since the range of $\mathcal{N} = 1$ gauge theories is much richer than that of $\mathcal{N} = 4$ SYM, it is worth rewriting the above AdS/CFT relations in terms of more general invariants of the

gauge theory. This may be elegantly done through anomaly matching, and in particular by making a connection through the holographic Weyl anomaly [14]. Note that a discussion of the $\mathcal{N} = 1$ SCFT description of the higher derivative theory was already given in [6], where special emphasis was placed on the technique of a-maximization. Here we wish to provide a more complete discussion of the relation between the gravity parameters G_5, g and c_2 and the gauge theory data.

3.1 The Weyl anomaly

For a four-dimensional field theory in a curved background, the Weyl anomaly may be parameterized by two coefficients, commonly denoted a and c (or equivalently b and b')

$$\langle T^\mu_\mu \rangle = \frac{c}{16\pi^2} C - \frac{a}{16\pi^2} E, \tag{3.4}$$

where

$$C = C^2_{\mu\nu\rho\sigma} = R^2_{\mu\nu\rho\sigma} - 2R^2_{\mu\nu} + \tfrac{1}{3}R^2 \tag{3.5}$$

is the square of the four-dimensional Weyl tensor, and

$$E = \widetilde{R}^2_{\mu\nu\rho\sigma} = R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2 \tag{3.6}$$

is the four-dimensional Euler invariant. At the two-derivative level, the holographic computation of the $\mathcal{N} = 4$ SYM Weyl anomaly gives $a = c = N^2/4$ [14]. Combining this with (3.3) then allows us to write

$$a = c = \frac{\pi L^3}{8G_5}, \tag{3.7}$$

which has the advantage of being completely general, independent of the particular gauge theory dual.

The prescription for obtaining the holographic Weyl anomaly for higher derivative gravity was worked out in [16, 35], and later extended in [36] for general curvature squared terms. The result is that, for an action of the form

$$e^{-1}\mathcal{L} = \frac{1}{2\kappa^2} \left(-R + 12g^2 + \alpha R^2 + \beta R^2_{\mu\nu} + \gamma R^2_{\mu\nu\rho\sigma} + \cdots \right), \tag{3.8}$$

the holographic Weyl anomaly may be written as [36]

$$g_{\mu\nu}\langle T^{\mu\nu} \rangle = \frac{2L}{16\pi G_5} \left[\left(-\frac{L}{24} + \frac{5\alpha}{3} + \frac{\beta}{3} + \frac{\gamma}{3} \right) R^2 + \left(\frac{L}{8} - 5\alpha - \beta - \frac{3\gamma}{2} \right) R^2_{\mu\nu} + \frac{\gamma}{2} R^2_{\mu\nu\rho\sigma} \right], \tag{3.9}$$

where L is related to g (to linear order) by

$$g = \frac{1}{L} \left[1 - \frac{1}{6L^2}(20\alpha + 4\beta + 2\gamma) \right]. \tag{3.10}$$

Comparison of (3.4) with (3.9) then gives the curvature-squared correction to (3.7)

$$a = \frac{\pi L^3}{8G_5} \left[1 - \frac{4}{L^2}(10\alpha + 2\beta + \gamma) \right]$$

$$c = \frac{\pi L^3}{8G_5} \left[1 - \frac{4}{L^2}(10\alpha + 2\beta - \gamma) \right]. \tag{3.11}$$

Turning now to the $\mathcal{N} = 2$ gauged supergravity Lagrangian of (3.1), we see that the curvature-squared corrections are proportional to the square of the five-dimensional Weyl tensor. This gives

$$(\alpha, \beta, \gamma) = \frac{c_2}{192}\left(\frac{1}{6}, -\frac{4}{3}, 1\right), \tag{3.12}$$

so that

$$a = \frac{\pi L^3}{8G_5}, \qquad c = \frac{\pi L^3}{8G_5}\left(1 + \frac{c_2}{24L^2}\right), \qquad g = \frac{1}{L}. \tag{3.13}$$

Note that the AdS radius is unshifted from that of the lowest order theory. This is because AdS is conformally flat, so that the Weyl-squared correction in (3.1) has no effect on the background. Finally, we may solve for c_2 to obtain

$$\frac{c_2}{24} = \frac{c - a}{ag^2}. \tag{3.14}$$

This is the key relation connecting the four-derivative terms in the gauged supergravity Lagrangian to the $\mathcal{N} = 1$ gauge theory data.

3.2 The R-current anomaly

A consistency check on the form of c_2 comes from the gravitational contribution to the anomalous divergence of the $U(1)_R$ current $\langle\partial_\mu(\sqrt{g}\,\mathcal{R}^\mu)\rangle$, since the latter is related by supersymmetry to the conformal anomaly $\langle T^\mu_\mu\rangle$.

The CFT U(1) anomaly is given by

$$\delta_I(\Lambda)Z_{\mathrm{CFT}} = \int \Lambda^I\left[\frac{\mathrm{tr}(G_I G_J G_K)}{24\,\pi^2}F^J \wedge F^K + \frac{\mathrm{tr}\,G_I}{192\,\pi^2}R_{ab} \wedge R^{ab}\right], \tag{3.15}$$

where G_I is a global $U(1)_I$ generator, and the trace is taken to be a sum over all the fermion loops. The AdS/CFT relation $Z_{\mathrm{CFT}} = \exp(-I_{\mathrm{bulk}})$ then connects this field theory anomaly to the coefficients of the Chern-Simons terms in the bulk supergravity:

$$I_{\mathrm{bulk}} = \cdots + \int\left[\frac{\mathrm{tr}\,(G_I G_J G_K)}{24\,\pi^2}A^I \wedge F^J \wedge F^K + \frac{\mathrm{tr}\,G_I}{192\,\pi^2}A^I \wedge R_{ab} \wedge R^{ab}\right], \tag{3.16}$$

where the ellipses denote the gauge invariant part of the action. Comparison to the $A \wedge R \wedge R$ term of (2.38) gives

$$\mathrm{tr}\,G_I = -\frac{\pi c_{2I}}{8G_5}. \tag{3.17}$$

To relate $c_2 \equiv c_{2I}\bar{M}^I$ to the central charges, we can use the relation [37, 38]

$$a = \frac{3}{32}\left(3\mathrm{tr}R^3 - \mathrm{tr}R\right), \quad c = \frac{1}{32}(9\mathrm{tr}R^3 - 5\mathrm{tr}R), \tag{3.18}$$

provided we can relate G_I appropriately to the U(1) charges R. A few comments are needed to explain how to identify the R-charge correctly. First of all, the R-charge is a particular linear combination of the G_I, proportional to $\bar{M}^I G_I$. Also, the supercharge Q_α should have R-charge one. The U(1) charges of Q_α can be read off from the coupling

between the gauge fields and the graviphoton in the gravity side, and the algebra is given by $[G_I, Q_\alpha] = P_I Q_\alpha$. This uniquely determines the R-charge as

$$R = \frac{\bar{M}^I G_I L}{P_I \bar{M}^I} \quad \rightarrow \quad \text{tr} R = -\frac{1}{P_I \bar{M}^I} \frac{\pi c_2 L}{8 G_5} \,. \tag{3.19}$$

Recall that the combination $P_I \bar{M}^I = 3/2$ can be determined from the vacuum solution, (2.35). By plugging this equation into (3.18), we obtain

$$\frac{c_2}{24} = \frac{8 G_5}{\pi L} (c - a) \,. \tag{3.20}$$

In addition, the gravitational constant also can be determined from the U(1) anomaly. Eq. (3.16) implies

$$\text{tr}(G_I G_J G_K) = \frac{\pi}{8 G_5} \left(12 c_{IJK} - \frac{g^2}{3} c_{(I} P_J P_{K)} \right) \,. \tag{3.21}$$

By multiplying $\bar{M}^I \bar{M}^J \bar{M}^K$ on both sides, we obtain

$$\frac{27}{8 L^3} \text{tr} R^3 = \frac{\pi}{8 G_5} \left(12 - \frac{3 c_2}{4 L^2} \right) \,. \tag{3.22}$$

The formula for the central charges (3.18) and (3.20) then gives

$$\frac{1}{16 \pi G_5} = \frac{a}{2 \pi^2 L^3} \,. \tag{3.23}$$

Using this relation, (3.20) can be rewritten as

$$\frac{c_2}{24 L^2} = \frac{c - a}{a} \,. \tag{3.24}$$

These results agree with those found through the holographic Weyl anomaly calculations, as expected for consistency.

3.2.1 Extracting the R-current anomaly from the $\mathcal{N} = 2$ case

Since the U(1) normalization may be somewhat obscure, we may perform an additional check by making contact with the $\mathcal{N} = 2$ SCFT literature. In fact, one can extract the c_2 result (3.14) from the analysis of [39], which studied R-symmetry anomalies in the $\mathcal{N} = 2$ SCFT dual to $\text{AdS}_5 \times S^5/\mathbb{Z}_2$. Of course, the appropriate supersymmetric CFT that is dual to our bulk $\mathcal{N} = 2$ AdS_5 theory has $\mathcal{N} = 1$ supersymmetry. Nevertheless, one can still use the analysis of [39], after carefully rewriting it in the language of $\mathcal{N} = 1$ anomalies. Before doing so, we will need to make a few general comments on the connection between the CFT R-current anomalies and the dual supergravity description.

The four-dimensional CFT R-current anomaly is sensitive to the amount of supersymmetry, and is given by [15]:

$$\partial_\mu (\sqrt{g} \, \mathcal{R}^\mu)_{\mathcal{N}=1} = \frac{c - a}{12 \pi^2} \tilde{R} R + \frac{5a - 3c}{9 \pi^2} \tilde{F} F \,, \tag{3.25}$$

$$\partial_\mu (\sqrt{g} \, \mathcal{R}^\mu)_{\mathcal{N}=2} = \frac{c - a}{4 \pi^2} \tilde{R} R + \frac{3(c - a)}{\pi^2} \tilde{F} F \,, \tag{3.26}$$

where F is the flux associated with the R-symmetry. The R-symmetry of $\mathcal{N}=2$ SCFTs is $\mathrm{U}(1)_R \times \mathrm{SU}(2)_R$. The $\mathrm{U}(1)_R$ symmetry of its $\mathcal{N}=1$ subalgebra is

$$\mathcal{R}_{\mathcal{N}=1} = \frac{1}{3}\mathcal{R}_{\mathcal{N}=2} + \frac{4}{3}I_3, \tag{3.27}$$

where I_1, I_2, I_3 are $SU(2)_R$ generators. The factor of $1/3$ in the relation above can also be seen in the gravitational contributions to $\partial_\mu(\sqrt{g}\,\mathcal{R}^\mu)$ in (3.25) and (3.26). Recall that the mixed U(1)-gravity-gravity anomaly $\partial_\mu(\sqrt{g}\,\mathcal{R}^\mu) \propto \tilde{R}R$ is represented in the bulk by the mixed gauge-gravity Chern-Simons interaction $\propto \int_{\mathrm{AdS}_5} A \wedge \mathrm{tr}(R \wedge R)$. Thus, the bulk CS term associated to the $\mathcal{N}=1$ SCFT will be $1/3$ of that corresponding to $\mathcal{N}=2$.

Furthermore, when using the results of [39], we will have to be careful with how the U(1) gauge field is normalized. In the AdS/CFT dictionary, the normalization of the gauge field kinetic term

$$S_{AdS_5} = \int d^4x \, dz \, \sqrt{-g} \, \frac{F_{\mu\nu}F^{\mu\nu}}{4\,g_{\mathrm{SG}}^2} \tag{3.28}$$

can be extracted by looking at the two-point function of the dual CFT currents sourced by the gauge field $A_\mu(\vec{x}) = A_\mu(\vec{x}, z)|_{\mathrm{boundary}}$. For a four-dimensional CFT, the general form of the two point function of such currents is given by [40]:

$$\langle J_i(x)J_j(y) \rangle = \frac{B}{(2\pi)^4}\left(\Box \delta_{ij} - \partial_i\partial_j \right)\frac{1}{(x-y)^4}\,, \tag{3.29}$$

where B is a numerical coefficient which is related to the normalization of the gauge kinetic term:

$$B \propto \frac{1}{g_{\mathrm{SG}}^2}\,. \tag{3.30}$$

For the $\mathcal{N}=2$ computation of [39] one finds $B=8$, while for the case of $\mathcal{N}=1$ super-symmetry [41] we read off $B=8/3$. Notice that the two results are again off by a factor of 3. We now have all the ingredients we need to apply the ($\mathcal{N}=2$ SCFT) analysis of [39] to our case ($\mathcal{N}=1$ SCFT). We have seen that both the gauge kinetic term normalization and the coefficient of the mixed gauge-gravity CS term will have to be adjusted.

The five-dimensional supergravity action of [39] takes the form

$$
\begin{aligned}
S &= \frac{N^2}{\pi^2 L^3}\int \sqrt{-g}\,\frac{F_{\mathcal{R}}^2}{4} + \frac{N}{16\pi^2 L}\int A^{\mathcal{R}} \wedge \mathrm{tr}(R \wedge R) \\
&= \frac{N^2}{4\pi^2 L^3}\int \left[\sqrt{-g}\,F_{\mathcal{R}}^2 - \frac{L^2}{16\,N}\epsilon_{\mu\nu\rho\lambda\sigma}A^\mu R^{\nu\rho\delta\gamma}R^{\lambda\sigma}{}_{\delta\gamma} \right],
\end{aligned}
\tag{3.31}
$$

where $A^{\mathcal{R}}$ is the gauge field that couples canonically to the R-current. This was the effective supergravity Lagrangian which was appropriate for comparison to the $\mathcal{N}=2$ SCFT. Since we are interested in comparing to a CFT with $\mathcal{N}=1$ SUSY, we will need to rescale both terms by appropriate factors of $1/3$:

$$S \to \frac{N^2}{4\pi^2 L^3}\int \left[\sqrt{-g}\,\frac{1}{3}F_{\mathcal{R}}^2 - \frac{L^2}{3\cdot 16\,N}\epsilon_{\mu\nu\rho\lambda\sigma}A^\mu R^{\nu\rho\delta\gamma}R^{\lambda\sigma}{}_{\delta\gamma} \right]\,. \tag{3.32}$$

Finally, we rescale the graviphoton, $A^{\mathcal{R}} = (\sqrt{3}/2)A$, to obtain a canonical gauge kinetic term:

$$S \to \frac{N^2}{4\pi^2 L^3} \int \left[\sqrt{-g}\ \frac{F^2}{4} - \frac{L^2}{32\sqrt{3}\,N} \epsilon_{\mu\nu\rho\lambda\sigma} A^\mu R^{\nu\rho\delta\gamma} R^{\lambda\sigma}{}_{\delta\gamma} \right] . \tag{3.33}$$

This is the action which should be compared to ours:

$$S_{\text{us}} = \frac{N^2}{4\pi^2 L^3} \int \sqrt{g} \left[-R - \frac{F^2}{4} + \frac{c_2}{24 \cdot 16\sqrt{3}} \epsilon_{\mu\nu\rho\lambda\sigma}\, A^\mu R^{\nu\rho\delta\gamma} R^{\lambda\sigma}{}_{\delta\gamma} + \dots \right] , \tag{3.34}$$

finally giving us

$$c_2 = \frac{12L^2}{N} = 24L^2 \frac{c-a}{a} , \tag{3.35}$$

in agreement with (3.14) and (3.24).

4 R-charged solutions

The embedding of the lowest order five-dimensional $\mathcal{N} = 2$ gauged $U(1)^3$ supergravity into IIB supergravity was done in [42]. If the three $U(1)$ charges are taken to be equal, we end up with the minimal supergravity system that we have considered above, (2.1). The static stationary non-extremal solutions are well known, and were found in [25]. For the truncation to minimal supergravity, they take the form

$$ds^2 = H^{-2} f dt^2 - H \left(f^{-1} dr^2 + r^2 d\Omega_{3,k}^2 \right)$$

$$A = \sqrt{\frac{3(kQ + \mu)}{Q}} \left(1 - \frac{1}{H} \right) dt, \tag{4.1}$$

where the metric functions H and f are:

$$H(r) = 1 + \frac{Q}{r^2},$$

$$f(r) = k - \frac{\mu}{r^2} + g^2 r^2 H^3 . \tag{4.2}$$

Here μ is a non-extremality parameter and $d\Omega_{3,k}^2$ for $k = 1, 0,$ or -1 corresponds to the unit metric of a spherical, flat, or hyperbolic 3-dimensional geometry, respectively.

4.1   Higher order corrected R-charged solutions

We wish to find corrections to the R-charged solutions (4.1) given the higher derivative Lagrangian (2.57). To this end, as in [43] we treat c_2 as a small parameter and expand the metric and gauge field as follows:

$$H(r) = 1 + \frac{Q}{r^2} + c_2 h_1(r) ,$$

$$f(r) = k - \frac{\mu}{r^2} + g^2 r^2 H^3 + c_2 f_1(r) ,$$

$$A = \sqrt{\frac{3(kQ + \mu)}{Q}} \left(1 - \frac{1 + c_2 a_1(r)}{H} \right) dt , \tag{4.3}$$

where h_1, f_1, and a_1 parameterize the corrections to the background geometry. Solving the equations of motion for the theory, we arrive at:

$$h_1 = -\frac{Q(kQ + \mu)}{72r^6H_0^2},$$

$$f_1 = \frac{-5g^2Q(kQ + \mu)}{72r^4} + \frac{\mu^2}{96r^6H_0},$$

$$a_1 = \frac{Q}{144r^6H_0^3}\left[4(kQ + \mu) - 3\mu - \frac{3Q\mu}{r^2}\right]. \tag{4.4}$$

The new corrected geometry is therefore given by

$$H(r) = H_0(r) + \frac{c_2}{24}\left[\frac{-Q(kQ + \mu)}{3r^6H_0^2}\right],$$

$$f(r) = f_0(r) + \frac{c_2}{24}\left[-\frac{8g^2Q(kQ + \mu)}{3r^4} + \frac{\mu^2}{4r^6H_0}\right],$$

$$A_t(r) = A_{t\,0}(r) - \frac{c_2}{24}\frac{\sqrt{3Q(kQ + \mu)}}{2r^8H_0^4}\left[2(kQ + \mu)r^2 - \mu r^2H_0\right], \tag{4.5}$$

where H_0, f_0, and A_0 refer to the background solutions (4.1) and (4.2). Finally, we should note that in the literature Q and μ are sometimes written in terms of a parameter β, defined by $\sinh^2\beta = kQ/\mu^2$.

We will state the $k = 0$ and $k = 1$ solutions explicitly, since they have several interesting applications: the former to studies of the hydrodynamic regime of the theory, and the latter to the issue of horizon formation for small black holes. For $k = 0$, the solution is given by

$$H(r) = H_0(r) + \frac{c_2}{24}\left[\frac{-Q\mu}{3\,r^6H_0^2}\right],$$

$$f(r) = f_0(r) + \frac{c_2}{24}\left[-\frac{8g^2\mu Q}{3r^4} + \frac{\mu^2}{4\,r^6H_0}\right],$$

$$A_t(r) = A_{t\,0}(r) - \frac{c_2}{24}\left[\frac{\sqrt{3Q\mu}}{2r^8H_0^4}\left(\mu r^2 - Q\mu\right)\right]. \tag{4.6}$$

while for $k = 1$ it is given by

$$H(r) = H_0(r) - \frac{c_2}{24}\left[\frac{Q(Q + \mu)}{3r^2(r^2 + Q)^2}\right],$$

$$f(r) = f_0(r) + \frac{c_2}{24}\left[-\frac{8g^2Q(Q + \mu)}{3r^4} + \frac{\mu^2}{4r^6H_0}\right],$$

$$A_t(r) = A_{t\,0}(r) - \frac{c_2}{24}\left[\frac{\sqrt{3Q(Q + \mu)}}{2r^8H_0^4}\left((2Q + \mu)r^2 - Q\mu\right)\right]. \tag{4.7}$$

4.2 Conditions for horizon formation

We would like to conclude this section with some comments on the structure of the horizon for the solutions that we have found. In particular, we are interested in whether higher derivative corrections will facilitate or hinder the formation of a horizon. In the standard

two-derivative theory, the BPS-saturated limit ($\mu = 0$) of the $k = 1$ solution (4.1)–(4.2) describes a geometry with a naked singularity, the so-called superstar [44]. Furthermore, even if the non-extremality parameter is turned on, one finds that a horizon develops only given a certain critical amount, $\mu \geq \mu_c$ [25]. It is therefore natural to ask what happens to such geometries once we start incorporating curvature corrections. For the superstar, we would like to see hints of horizon formation. In the non-extremal case, on the other hand, it would be nice to determine whether the inclusion of higher-derivative corrections leads to a smaller (larger) critical value μ_c, increasing (decreasing) the parameter space for the appearance of a horizon. However, one should keep in mind that our arguments are only suggestive, since our analysis is perturbative, while the formation of a horizon is a non-perturbative process. Moreover, given that even in the non-extremal case turning on μ does not guarantee the presence of a horizon, it is not clear at all whether higher derivative corrections can be enough to push the superstar to develop a horizon. A more proper analysis would involve looking directly at the SUSY conditions, and asking whether they are compatible with having a superstar solution with a finite horizon. In fact, there are already studies which seem to indicate [45] that this may not be possible.

The spherically symmetric solutions presented in (4.7) are of the form:

$$ds^2 = F_1(r)\,dt^2 - F_2(r)\,dr^2 - F_3(r)\,d\Omega_3^2\,. \tag{4.8}$$

Horizons appear at zeroes of the function $F_1(r)$. One can make arguments about their existence without having to solve explicitly for their exact location. Notice that $F_1(r)$ is a positive function for large r. Thus, a sufficient condition for having at least one horizon is

$$F_1(r_{\min}) \leq 0\,, \tag{4.9}$$

where $r_{\min}$ is a (positive) minimum of $F_1(r)$. This was the reasoning used in [25] to study the properties of the horizon of the non-extremal solution.

For the corrected superstar solution we have, expanding in c_2:

$$F_1 \equiv \frac{f}{H^2} = \frac{f_0 + c_2\left(f_1 - 2f_0 h_1 H_0^{-1}\right)}{H_0^2} + \mathcal{O}\left(c_2^2\right)\,. \tag{4.10}$$

It is easy to see that, to leading order, the numerator does not vanish. With the inclusion of higher-derivative terms, however, it picks up a negative contribution, hinting at the possibility of a horizon. Furthermore, the minimum of the function $F \equiv f_0 + c_2(f_1 - 2f_0 h_1 H_0^{-1})$ will shift. Let's see precisely how that happens. To lowest order, its minimum is given by $x_{\min}^{(0)} = 2Q$, which in turn gives us $F(x_{\min}^{(0)}) = 1 + 27g^2Q/4$. Including higher order corrections, we find

$$x_{\min} = x_{\min}^{(0)} + c_2 x_{\min}^{(1)} = 2Q - c_2 \frac{81g^2Q - 4}{4374Qg^2}\,. \tag{4.11}$$

Now we have

$$F(x_{\min}) = 1 + 27g^2Q/4 + c_2\left(\frac{1}{972Q} - \frac{g^2}{48}\right)\,, \tag{4.12}$$

which tells us that the minimum of the function will be slightly closer to zero as long as $g^2 Q > 4/81$.

The analysis of the conditions for the existence of a horizon in the non-extremal case ($\mu \neq 0$) is significantly more involved. The expression for the corrected horizon radius in terms of the original, two-derivative horizon radius r_0 is:

$$r_H = r_0 \left(1 + \frac{c_2}{24} \left\{ \frac{g^4 H_0^4 \left(3Q^2 - 26Qr_0^2 + 3r_0^4 \right) - 2g^2 H_0^2 \left(13Q - 3r_0^2 \right) + 3}{24 H_0 r_0 \left[g^2 H_0^2 \left(Q - 2r_0^2 \right) - 1 \right]} \right\} \right) . \quad (4.13)$$

Notice that we traded μ in favor or r_0 in the expression above by making use of $f_0(r_0) = 0$, i.e. the relation $\mu/r_0^2 = 1 + g^2 r_0^2 H_0^3$. As we mentioned above, in the two-derivative case one finds a critical value μ_{crit} above which a horizon will form. It would certainly be interesting to explore for which parameter values r_H decreases or increases, and more importantly, how the (corrected) critical value of μ is affected by the curvature corrections. We leave this to future studies.

5 Thermodynamics

We may now study some of the basic thermodynamic properties of the non-extremal solutions constructed above. With an eye towards AdS/CFT in the Poincaré patch, we will focus on the $k = 0$ solution (4.6), although the analysis may easily be carried out for the other cases as well. We begin with the entropy, which for Einstein gravity is characterized by the area of the event horizon. In the presence of higher derivative terms, however, this relation is modified, and the entropy is no longer given by the area law. Instead, we may turn to the Noether charge method developed in [46] (see also [47, 48]).

The original Noether charge method is only applicable to a theory with general covariance, but has been extended to a theory with gravitational Chern-Simons terms in [49]. Our action includes a mixed Chern-Simons term of the form $A \wedge R \wedge R$. But as long as we keep this term as it is, with a bare gauge potential, the general covariance is unbroken and we can still use the original formulation. In the absence of covariant derivatives of the Riemann tensor, the entropy formula is given by [46]

$$S = -2\pi \int_{\Sigma} d^{D-2}x \sqrt{-h} \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma} , \quad (5.1)$$

where Σ denotes the horizon cross section, h is the induced metric on the it and $\epsilon_{\mu\nu}$ is the binormal to the horizon cross section.

For the metric ansatz (4.1) the only non-vanishing component of the binormal $\epsilon_{\mu\nu}$ is

$$\epsilon_{\text{tr}} = -\epsilon_{\text{rt}} = H^{-1/2} . \quad (5.2)$$

Applying the prescription (5.1) to the action (2.57), we obtain, to linear order in c_2,

$$S = \frac{A}{8G_5} \left[-g^{\mu\rho} g^{\nu\sigma} + \frac{c_2}{24} \left(-\frac{1}{4} C^{\mu\nu\rho\sigma} - \frac{1}{32} g^{\mu\rho} g^{\nu\sigma} F^2 + \frac{5}{12} g^{\nu\sigma} F^{\mu\lambda} F^{\rho}{}_{\lambda} - \frac{1}{16} F^{\mu\nu} F^{\rho\sigma} \right) \right] \epsilon_{\mu\nu} \epsilon_{\rho\sigma} \Big|_{r=r_+}$$

$$= \frac{A}{4G_5} \left[1 + c_2 \frac{\mu \left(Q + 3r_0^2 \right)}{48 \left(r_0^2 + Q \right)^3} \right] , \quad (5.3)$$

where $A = \int \sqrt{-h}\, d\Omega_{3,0}$ is the area of the horizon for the solution to the higher derivative theory. Also, r_+ denotes the radius of the event horizon for the corrected black brane solution, while r_0 is the horizon location for the original, two-derivative solution (4.2). The former can be found by requiring that the $g_{tt} = f(r)/H(r)^2$ component of the corrected metric vanishes.[7] Similarly, r_0 satisfies $f_0(r_0) = 0$. Notice that the non-extremality parameter μ can be expressed entirely in terms of r_0 and Q:

$$f_0(r_0) = 0 \;\; \Rightarrow \;\; \mu = \frac{g^2(r_0^2 + Q)^3}{r_0^2}\,. \tag{5.4}$$

We can therefore eliminate μ from (5.3), and write the entropy in the following form:

$$S = \frac{A}{4G_5}\left[1 + c_2 g^2 \frac{Q + 3r_0^2}{48\, r_0^2}\right]\,. \tag{5.5}$$

The first term above is simply the contribution coming from the area, while the remaining $\mathcal{O}(c_2)$ term is the expected deviation from the area law.

In order to arrive at the entropy density, we need one more ingredient, which is the relation between the corrected and uncorrected horizon radii r_+ and r_0:

$$r_+ = r_0 \left(1 + \frac{c_2}{24}\frac{g^2(r_0^2 + Q)(3Q^2 - 26Qr_0^2 + 3r_0^4)}{24r_0^4(Q - 2r_0^2)}\right)\,. \tag{5.6}$$

This is because the area A appearing in (5.5) is computed using r_+. This expression allows us to write the entropy per unit three-brane spatial volume entirely in terms of r_0 as well as the physical parameters of the theory

$$\begin{aligned}
s &= \frac{(r_0^2 + Q)^{3/2}}{4G_5 L^3}\left(1 + \frac{c_2}{24}\frac{g^2(3Q^2 - 14Qr_0^2 - 21r_0^4)}{8r_0^2(Q - 2r_0^2)}\right) \\
&= \frac{2(r_0^2 + Q)^{3/2}}{\pi L^6}\left(a + (c - a)\frac{3Q^2 - 14Qr_0^2 - 21r_0^4}{8r_0^2(Q - 2r_0^2)}\right)\,.
\end{aligned} \tag{5.7}$$

In the second line we have used the relations (3.13) to replace the gravitational quantities G_5 and c_2 by the central charges of the dual CFT. Notice that the lowest order term above matches the two-derivative entropy computation of [50].

While r_0 is the coordinate location of the horizon in the lowest order computation, it is not in itself a physically relevant parameter. Instead, it may be viewed as a proxy for the Hawking temperature associated with the non-extremal solution. A simple way of computing this temperature is to identify it with the inverse of the periodicity of Euclidean time τ. The relevant components of the metric are given by

$$ds^2 = H^{-2}f d\tau^2 + Hf^{-1}dr^2 + \cdots\,, \tag{5.8}$$

and the horizon is located at $f(r_+) = 0$. Expanding near the horizon and identifying the proper period of τ to remove the conical singularity yields the temperature

$$T_H = \frac{(r_0^2 + Q)^{1/2}}{2\pi L^2}\left[\frac{(2r_0^2 - Q)}{r_0^2} + \frac{c_2}{24L^2}\frac{(3Q^3 + 4Q^2r_0^2 + 59Qr_0^4 - 10r_0^6)}{8r_0^4(2r_0^2 - Q)}\right]\,. \tag{5.9}$$

---

[7]To linear order in the expansion parameter c_2, this coincides with demanding that $f(r)$ vanishes.

In principle, we may invert this expression to obtain r_0 as a function of temperature T_H and charge Q. This then allows us to rewrite the entropy density as a function of charge and temperature, $s = s(T_H, Q)$. In practice, however, non-trivial R-charge introduces a new scale, so that the entropy density/temperature relation no longer takes the simple form $s \sim T^3$ resulting from simple dimensional analysis.

6 Discussion

The main result of the previous section is the derivation of the entropy (5.7) of an R-charged black brane including higher-derivative corrections, which are controlled by the parameter c_2. Furthermore, the identification of the gravitational parameters G_5 and c_2 in terms of the central charges a, c of the dual CFT has allowed us to express the entropy in terms of microscopic, gauge theory data. In particular, the relation between c_2, which signals the contribution coming from R^2 terms, and the CFT central charges is given by $c_2 = 24L^2(c - a)/a$.

A non-trivial check on the corrections to the entropy can be done by considering the zero R-charge ($Q = 0$) limit of (5.7), which should agree with the analysis of [19]. For a Lagrangian of the form

$$\mathcal{L} = \frac{R}{16\pi G_5} - \Lambda + \alpha_1 R^2 + \alpha_2 R_{\mu\nu}^2 + \alpha_3 R_{\mu\nu\rho\sigma}^2\,, \tag{6.1}$$

the authors of [19] showed that the entropy density of a 5D AdS black brane solution is given by

$$s = \frac{2\pi}{L^3 z_0^3}\left[\frac{1}{8\pi G_5} - \frac{18}{L^2}(5\alpha_1 + \alpha_2) + \frac{12}{L^2}\alpha_3\right]\,, \tag{6.2}$$

where L denotes the AdS curvature radius, $L^2 = -6/(8\pi G_5 \Lambda)$, and z_0 is the leading order horizon location in the coordinates used in [19]; in our notation, $z_0 = 1/r_0$. Comparing (6.1) to our action, where the only curvature corrections that survive the $Q = 0$ limit come in the form of $C_{\mu\nu\rho\sigma}^2 = \frac{1}{6}R^2 - \frac{4}{3}R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2$, we read off:

$$16\pi G_5 \alpha_1 = \frac{1}{48}\frac{c_2}{24}\,, \quad 16\pi G_5 \alpha_2 = -\frac{1}{6}\frac{c_2}{24}\,, \quad 16\pi G_5 \alpha_3 = \frac{1}{8}\frac{c_2}{24}\,. \tag{6.3}$$

Making use of these expressions, the entropy of [19] takes the form

$$s = \frac{1}{4L^3 z_0^3 G_5}\left[1 + \frac{21}{16}\frac{c - a}{c}\right]\,, \tag{6.4}$$

matching nicely the $Q = 0$ limit of (5.7), as expected.

We should point out that a similar discussion has appeared very recently in [18], where the authors considered the hydrodynamic regime of the CFT dual to the zero R-charge black brane background of [19]. In [18], however, higher derivative corrections associated with R^2 and $R_{\mu\nu}$ are eliminated via a field redefinition, making direct comparison to our entropy less straightforward.

Our interest in studying higher order corrections to R-charged AdS_5 black holes is also motivated by our desire to investigate corrections to the hydrodynamic regime of the dual

theory. It is natural to apply the results of this work to the calculation of η/s, the shear viscosity to entropy ratio, which has recently received a great deal of attention. In particular, our present construction of higher-derivative corrected R-charged black holes allows for a generalization of the finite coupling shear viscosity calculation to the case of finite (R-charge) chemical potential. This is an avenue which we are currently exploring [51].

We would like to conclude with a few comments on the issue of horizon formation. As we mentioned in section 3, the so-called superstar solution at the two-derivative level has a naked singularity. With the inclusion of higher derivative contributions, it appears that the corrected superstar may develop a horizon, provided that the charges are large enough, $g^2Q > 4/81$. However, we should note that our analysis is entirely perturbative, while horizon formation is an intrinsically non-perturbative phenomenon. While our results show that the first corrections to the geometry seem to push the superstar solution "in the right direction," increasing the chances of forming a horizon, a more rigorous analysis is certainly needed to reach a conclusive result.

Acknowledgments

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